

Galois Representations and Automorphic Forms

Book of abstracts

August 20-26, 2023

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Abstracts

Multiple base discrete logarithm and wild 1-motives

Dorota Blinkiewicz

Adam Mickiewicz University in Poznań, Poland

Tuesday
August 22
16:40 -
17:10

A multiple base (g_1, \dots, g_n) discrete logarithm problem in an abelian group G concerns solutions of $g_1^{x_1} \cdot \dots \cdot g_n^{x_n} = g$ in $(x_1, \dots, x_n) \in \mathbb{Z}^n$, where $g_1, \dots, g_n, g \in G$. During the lecture we show that the local to global multiple base discrete logarithm problem in the group $\mathbb{G}_m^2(\mathcal{O}_{F,S}) = (\mathcal{O}_{F,S}^\times)^2$, for a number field F , has counterexamples for arbitrary big bases, including infinite bases.

Based on this result we present families of pairwise non-isomorphic Deligne 1-motives, over rings $\mathcal{O}_{F,S}$, which have the same reductions to torsion 1-motives for all $v \notin S$. This is joint work with G. Banaszak.

Anticyclotomic CM Iwasawa theory at inert primes

Ashay Burungale

The University of Texas at Austin, USA

Thursday
August 24
15:10 -
16:00

The talk plans to describe aspects of Iwasawa theory of a CM elliptic curve defined over \mathbb{Q} along the anticyclotomic \mathbb{Z}_p -tower of the CM field for p inert. The theory was envisaged by K. Rubin in the mid 80's. (Joint with S. Kobayashi and K. Ota)

On Kolyvagin's conjecture and its refinement

Francesc Castella

University of California, Santa Barbara, USA

Friday
August 25
10:40 -
11:30

Let E/\mathbb{Q} be an elliptic curve, and $p > 2$ a prime of good ordinary reduction. In 1991, Kolyvagin conjectured the non-triviality of a system of cohomology classes derived from Heegner points on E of varying conductors. The first major result towards Kolyvagin's conjecture is due to W. Zhang, who obtained a proof of the conjecture under certain ramification hypotheses on $E[p]$. In this talk, I will explain a new proof of Kolyvagin's conjecture building on Iwasawa theoretic techniques and the work of Cornut-Vatsal. Our result treats the cases where $E[p]$ is irreducible as a Galois module (with no ramification hypotheses) as well as the first cases where E admits a rational p -isogeny. Moreover, by the same methods we also prove a refinement of Kolyvagin's conjecture posed by W. Zhang in 2014. Based on a joint work with A. Burungale, G. Grossi, and C. Skinner.

On Emerton's factorisation of completed cohomology

Pierre Colmez
CNRS, Sorbonne Université, France

Wednesday
August 23
10:40 -
11:30

I will explain a new proof of Emerton's factorisation of the completed cohomology of the tower of modular curves. This is joint work with Shanwen Wang.

Generalised Hecke eigenvalues

Henri Darmon
McGill University, Canada

Friday
August 25
9:30 -
10:20

In situations where Hecke operators act non semi-simply on spaces of modular forms, the resulting generalized eigenvalues can encode rich arithmetic information. In the case of p -adic modular forms of weight one, they hold a possible key to explicit class field theory for certain non CM fields. The setting of spaces of mod p modular forms of weight two, where they are closely related to the local behaviour of certain global cohomology classes constructed by M. Flach, will be discussed in this talk. This is an account of work in progress with Alice Pozzi.

Parahoric level p -adic L -functions for automorphic representations with Shalika models

Mladen Dimitrov
University of Lille, France

Wednesday
August 23
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17:30

t.b.a.

On a local-global principle for quadratic twists of abelian varieties

Francesc Fité Naya
Universitat de Barcelona, Spain

Tuesday
August 22
15:10 -
16:00

Let A and A' be abelian varieties defined over a number field k . In the talk I will consider the following question: Is it true that A and A' are quadratic twists of one another if and only if they are quadratic twists modulo p for almost every prime p of k ? Serre and Ramakrishnan have given a positive answer in the case of elliptic curves and a result of Rajan implies the validity of the principle when the endomorphism ring of A (and hence also that of A') over an algebraic closure of \mathbb{Q} is just \mathbb{Z} . For not necessarily simple abelian varieties, I will show that the answer is affirmative up to dimension 3, but that it becomes negative in dimension 4. Time permitting, I will present ongoing joint work with E. Ambrosi that leads to more general results.

Counting local system with specified monodromy

Yuval Flicker
Ariel University, Israel

Tuesday
August 22
12:10 -
13:00

I shall try to describe an explicit formula for the cardinality of a set of Galois-invariant isomorphism classes of irreducible rank two ℓ -adic smooth sheaves (= local systems) on $X_1 - S_1$, where X_1 is a smooth projective absolutely irreducible curve of genus g over a finite field \mathbb{F}_q , and S_1 is a reduced divisor, with pre-specified monodromy data at S , and how to obtain it. The approach is based on translating the question to one concerning automorphic representations with corresponding ramification, then applying the trace formula with suitably chosen pseudo-coefficients to capture the ramification.

Triviality of the Hecke action on ordinary Drinfeld cuspforms of level $\Gamma_1(t^n)$

Shin Hattori
Tokyo City University, Japan

Friday
August 25
12:10 -
13:00

Let p be a rational prime, $q > 1$ a p -power integer and \wp a non-constant irreducible polynomial in $\mathbb{F}_q[t]$. The notion of Drinfeld modular form is an analogue over $\mathbb{F}_q(t)$ of that of elliptic modular form. The expectation is that there should be a deep \wp -adic theory of Drinfeld modular forms which is comparable to the elliptic analogue. However, \wp -adic properties of Drinfeld modular forms are less well understood than the p -adic elliptic case, and sometimes classical methods do not work or yield nothing useful.

In this talk, as an example of such unusual phenomena, I will explain that all Hecke operators act trivially on the space of ordinary Drinfeld cuspforms of level $\Gamma_1(t^n)$ for any positive integer n . It gives a counterexample to Gekeler's question, which asks if the weak multiplicity one holds for Drinfeld modular forms when the weight is fixed.

Adjoint L-value formula and the Tate conjecture for Shimura surfaces over \mathbf{Q}

Haruzo Hida
University of California, Los Angeles, USA

Monday
August 21
9:30 -
10:20

Let f be an elliptic Hecke eigen form with two dimensional Galois representation ρ_f . Let ρ_f act on the three dimensional trace = 0 subspace of the 2 by 2 matrices by conjugation and write this representation as $Ad(\rho_f)$. I first present an explicit formula of the adjoint L-value $L(1, Ad(\rho_f) \otimes \chi_E)$ for a real quadratic field E with associated quadratic character χ_E . Then I choose an indefinite quaternion algebra D over \mathbf{Q} which ramifies only at split primes in E/\mathbf{Q} and put $D_E = D \otimes_{\mathbf{Q}} E$. Let Sh_E be the Shimura surface associated to D_E^\times which is defined over its reflex field \mathbf{Q} . We describe a way to prove that $H^0(K, H_{\text{et}}^2(Sh_E, \mathbf{Q}_l(1)))$ is generated by Shimura subcurves of Sh_E coming from all quaternion subalgebras $B \subset D_E$ over \mathbf{Q} . Here K is any abelian extension of \mathbf{Q} . If time allows, we sketch how to generalize the argument to all quaternionic Shimura varieties over a totally real field F .

On the Universal Deformation Ring of Residual Galois Representations with Three Jordan Holder Factors

Xiaoyu Huang

City University of New York, USA

Thursday
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17:10

In this work, we study Fontaine-Laffaille, essentially self-dual deformations of a mod p non-semisimple Galois representation of dimension n with its Jordan-Holder factors being three mutually non-isomorphic absolutely irreducible representations. We show that under some conditions on certain Selmer groups, the universal deformation ring is a discrete valuation ring. Given enough information on the Hecke side, we also prove an $R = T$ theorem. We then apply our results to abelian surfaces with cyclic rational isogenies and to certain 6-dimensional representations arising from automorphic forms congruent to Ikeda lifts. In particular, our result identifies the special L -value conditions for the uniqueness of the abelian surface isogeny class, and assuming the Bloch-Kato conjecture, an $R = T$ theorem for the 6-dimensional representations.

Ribet's lemma in residually indistinguishable case

Mahesh Kakde

Indian Institute of Science, India

Thursday
August 24
12:10 -
13:00

Ribet's method, and its refinements by Wiles, is the only systematic method of producing non-trivial elements in Selmer groups. If χ_1 and χ_2 are two characters from a compact group into a complete local noetherian local ring R , then Ribet's lemma gives a strategy of constructing an extension of χ_1 by χ_2 . An important assumption that occurs in Ribet's lemma is that χ_1 and χ_2 are distinct modulo the maximal ideal of R . In this talk I will give a formulation of Ribet's Lemma without this assumption and sketch its proof. This is joint work with Samit Dasgupta, Jesse Silliman and Jiuya Wang.

Group actions with vanishing cohomology

Kiran Kedlaya

University of California San Diego, USA

Tuesday
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10:40 -
11:30

For G a group, H a subgroup, and M a G -module, we consider the question of when the vanishing (or finite dimensionality) of all H -cohomology groups of M implies the same for all G -cohomology groups. For example, this follows from the Hochschild-Serre spectral sequence when H is subnormal in G , but can fail otherwise.

Motivated by a result of Kostant in the classical theory of Lie groups, we consider this question in the special case where G is a p -adic Lie group, H is a nilpotent group, and M is an analytic G -module. The picture is mostly conjectural except in the case where H is procyclic and M is of positive characteristic, in which case "analytic" must be replaced by the "super-Holder" condition recently introduced by Berger-Rozenstajn; the result in this case is joint with Annie Carter.

A structure result for vexing Hecke algebras

Jaclyn Lang
Temple University, USA

Monday
August 21
15:10 -
16:00

Fix a weight 2 CM modular form f with trivial character and level p^2 for some prime p . How many eigenforms of the same weight and level are congruent to f modulo a prime l ? We will sketch a proof that when p is $-1 \pmod{l}$, this number is always divisible by l . Such an f is an example of a modular form that is “vexing at $p \pmod{l}$ ”, and the l -divisibility phenomenon is true for all such vexing forms (if l is at least 5). These vexing forms are so named due to the difficulties they pose in modularity lifting theorems. The divisibility result is deduced from a structure result on the corresponding Hecke algebra: we show that it is free over the group ring of a cyclic p -group. Our techniques use both modular representation theory as well as geometric/cohomological methods. This is joint work in progress with Robert Pollack and Preston Wake.

p -adic L -functions for $GSp(4) \times GL(2)$

Zheng Liu
University of California, Santa Barbara, USA

Wednesday
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12:10 -
13:00

For a cuspidal automorphic representation Π of $GSp(4)$ and a cuspidal automorphic representation π of $GL(2)$, Furusawa’s formula can be used to study the special values of the degree-eight p -adic L -function $L(s, \Pi \times \pi)$. In this talk, I will explain a construction of the p -adic L -function for $\Pi \times \pi$ by using Furusawa’s formula and a family of Eisenstein series.

A new main conjecture in noncommutative Iwasawa theory

Andreas Nickel
Universität der Bundeswehr München, Germany

Monday
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17:30

We formulate a new equivariant main conjecture in the Iwasawa theory of number fields and study its properties. More precisely, we do this for arbitrary one-dimensional p -adic Lie extensions of an arbitrary number field K . So we do not require the extension to be abelian or K to be totally real. We discuss its relation to existing conjectures and deduce its validity in several interesting cases. This is joint work with Antonio Mejías Gil.

Duality in p -adic pro-étale cohomology of analytic spaces

Wiesława Nizioł
CNRS, Sorbonne Université, France

Monday
August 21
12:10 -
13:00

I will discuss duality theorems in p -adic pro-étale cohomology of analytic spaces. This is based on a joint work with Pierre Colmez and Sally Gilles.

Predicting slopes of modular forms and reductions of crystalline representations

Robert Pollack
Boston University, USA

Thursday
August 24
10:40 -
11:30

The recently proven ghost conjecture predicts slopes of modular forms in the so-called regular case.

This regularity condition is nearly that the associated residual representation is reducible. However, slopes in the locally irreducible case still remain mysterious. In this talk, after reviewing the regular ghost conjecture, we will examine the irregular case and discuss recent progress towards formulating a conjecture in this situation. The story remains incomplete, but we will discuss how an irregular ghost conjecture is intimately related to reductions of crystalline representations. This is a joint project with John Bergdall.

The center of the category of smooth representations

Peter Schneider
Universität Münster, Germany

Wednesday
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9:30 -
10:20

Let G be the group of F -points of a connected reductive group over a p -adic field F , and let $Mod_k(G)$ denote the category of smooth G -representations over a field k . The center of $Mod_k(G)$ is the ring of natural endomorphisms of the identity functor on $Mod_k(G)$. It was one of the big achievements of Bernstein around 1990 to compute explicitly this center in the case where k is the field of complex numbers. The point here is that this center is very big even if the group G has a trivial center $Z(G)$. In contrast assume now that k has characteristic p . Then in joint work with Ardakov we showed in 2021 that the center of $Mod_k(G)$ is a certain completion of the group ring $k[Z(G)]$. As implicit already in Bernstein's work and explicitly suggested in 2019 by Dotto/Emerton/Gee the center of an abelian category should be a ringed space (or site) whose ring of global sections is the "naive" center. Again with Ardakov we showed in 2022 the existence of such a "central" space for any Grothendieck abelian category, and we worked this out for $Mod_k(SL_2(\mathbb{Q}_p))$.

Katz' p -adic L function for CM fields and Stark's conjecture

Ehud de Shalit
Hebrew University, Israel

Tuesday
August 22
9:30 -
10:20

In this talk I want to revisit some unpublished work of mine from 1986 (!). In the 1970's Katz constructed p -adic L functions for Hecke characters of quadratic imaginary fields by p -adic interpolation of real analytic Eisenstein series. Coates and Wiles gave an alternative construction based on the Euler system of elliptic units. Over arbitrary CM fields, Katz' approach generalizes, but the analogous Euler systems of "abelian units" are missing. In our talk we shall explore the hypothetical relation between the two constructions. A surprising feature is that it is possible to say a-priori what should be the periods involved, and the fact that they are equal amounts to a p -adic generalization of Shimura's period relations, which is a consequence of a theorem of Blasius from 1994. We shall formulate a p -adic conjecture related to such a hypothetical construction of Katz' p -adic L function, and explain its relation to Stark's conjecture.

Integral period relations and congruence numbers

Jacques Tilouine
Universite Paris 13, France

Monday
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11:30

In an earlier work with E. Urban, we established integral period relations for the base change of a classical cusp form to a quadratic extension. In this talk, we consider other cases of Langlands functoriality for which we formulate conjectural integral period relations. In several works in progress, we prove partial results towards these.

Rational torsion in modular Jacobians

Preston Wake
Michigan State University, USA

Thursday
August 24
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10:20

For a prime number N , Ogg's conjecture states that the torsion in the Jacobian of the modular curve $X_0(N)$ is generated by the cusps. Mazur proved Ogg's conjecture as one of the main theorems in his "Eisenstein ideal" paper. I'll talk about a generalization of Ogg's conjecture for squarefree N and a proof using the Eisenstein ideal. This is joint work with Ken Ribet.

Critical Lambda-adic modular forms

Carl Wang-Erickson
University of Pittsburgh, USA

Wenesday
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16:00

Coleman proved that the dimension of spaces of overconvergent modular forms of critical slope is locally constant as a function of its integer weight with respect to the standard p -adic topology. While this suggests that critical slope overconvergent forms could appear in p -adic families parameterized by weight, as in the ordinary case, this cannot happen in the eigencurve due to straightforward topological restrictions. In this talk we will discuss a construction of p -adic families of critical slope overconvergent modular forms, interpolating "ordinary avatars" using techniques of Hida theory. We also explain applications to the corresponding Galois representations. This is joint work with Francesc Castella.

Speakers

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List of Participants

Grzegorz Banaszak – Adam Mickiewicz University, Poznań, Poland
Stefan Barańczuk – Adam Mickiewicz University, Poznań, Poland
Tobias Berger – University of Sheffield, UK
Adel Betina – University of Copenhagen, Denmark
Dorota Blinkiewicz – Adam Mickiewicz University, Poznań, Poland
Ashay Burungale – The University of Texas at Austin, USA
Yuzhen Cao – McGill University, Canada
Francesc Castella – University of California, Santa Barbara, USA
Tymoteusz Chmiel – Jagiellonian University, Poland
Pierre Colmez – CNRS, Sorbonne Université, France
Henri Darmon – McGill University, Canada
Mladen Dimitrov – University of Lille, France
Francesc Fité – Universitat de Barcelona, Spain
Yuval Flicker – Ariel University, Israel
Ho Leung Fong – University of Sheffield, UK
Filip Gawron – Jagiellonian University, Poland
Shin Hattori – Tokyo City University, Japan
Haruzo Hida – University of California, Los Angeles, USA
Xiaoyu Huang – City University of New York, USA
Tomasz Jędrzejak – University of Szczecin, Poland
Mahesh Kakde – Indian Institute of Science, India
Kiran Kedlaya – University of California San Diego, USA
Ghizlane Kettani – Paris University, France
Chan-Ho Kim – Korea Institute for Advanced Study, Republic of Korea
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Preston Wake – Michigan State University, USA
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Wojtek Wawrów – London School of Geometry & Number Theory, UK
Richard Yi – UC San Diego, USA