Algebraic K-Theory and Arithmetic

Book of abstracts

July 7-13, 2024

Contents

Abstracts

Functoriality of the p-adic Simpson correspondence by proper push forward

Tuesday
July 9
9:30 -
10:20

Ahmed Abbes

CNRS Laboratoire Alexander Grothendieck, France

Faltings initiated in 2005 a p-adic analogue of the (complex) Simpson correspondence whose construction has been taken up by various authors, according to several approaches. After recalling the one initiated by myself with Michel Gros, I will present our initial result on the functoriality of the p-adic Simpson correspondence by proper push forward, leading to a generalization of the relative Hodge-Tate spectral sequence. If time permits, I will give a brief overview of an ongoing project with Michel Gros and Takeshi Tsuji, aimed at establishing a more robust framework for achieving broader functoriality results of the p-adic Simpson correspondence, by both proper push forward and pullback.

Tame fundamental groups of rigid spaces

Monday
July 8
10:10 -
11:00

Piotr Achinger

IMPAN Warszawa, Poland

p-adic analytic objects such as the affinoid unit disc often have complicated fundamental groups. In order to deal with this difficulty, we introduce the notion of a tamely ramified finite etale cover of a rigid analytic space, defined in terms of its ramification at the points of Huber's "universal compactification". The resulting "tame fundamental group" is expected to have good finiteness properties. Our main result is that it is (topologically) finitely generated (for a quasi-compact and quasi-separated rigid space over an algebraically closed field). The proof uses logarithmic geometry beyond its usual scope of finitely generated monoids to (eventually) reduce the problem to the more classical one of finite generation of tame fundamental groups of algebraic varieties over the residue field. This is joint work with Katharina Hübner, Marcin Lara, and Jakob Stix.

K-Theory of Adic Spaces

Friday
July 12
11:00 -
11:50

Grigory Andreychev
Institute for Advanced Study, Princeton, USA

We will explain a new approach to the K-theory of analytic adic spaces via condensed mathematics. Its main advantage is that it allows us to define K-theory in that context as the (continuous) K-theory of a certain dualizable stable ∞-category associated to an analytic adic space (as opposed to previous constructions that produced the K-theory spectrum somewhat indirectly). In particular, it makes the study of its general properties much more feasible; for instance, it is possible to prove a very general descent statement without any restrictive hypotheses on the adic space. Moreover, one can prove an analog of the GRR theorem in this context.

$\widehat{\mathcal{D}}$-modules on rigid analytic spaces: recent progress

Thursday
July 11
10:10 -
11:00

Konstantin Ardakov
University of Oxford, UK

$\widehat{\mathcal{D}}$-modules on rigid analytic spaces were introduced by Wadsley and myself over ten years ago in order to relate locally analytic representations of semisimple p-adic Lie groups to p-adic differential equations through a version of the Beilinson-Bernstein localisation theorem on rigid analytic flag varieties. I will give a survey of recent works in this area, including the six-functor formalisms of Bode, Rodriguez-Camargo and Soor, and the p-adic Riemann Hilbert correspondence for $\widehat{\mathcal{D}}$-modules due to Wiersig.

Log prismatic cohomology, motivic sheaves and slice filtrations

Friday
July 12
15:10 -
16:00

Federico Binda
University of Milan, Italy

In this talk I will explain how to construct (filtered) $\mathbb{P}^1$-spectra in the stable log motivic homotopy category $\mathrm{logSH}(S)$ representing (log) prismatic and syntomic cohomology for S a quasi-syntomic scheme, and how to deduce from this some structural properties of such cohomology theories (like Gysin sequences and blow-up formulas), as well as comparison theorems. If time permits, I will explain how to use these results to establish a natural compatibility between the analogue of Voevodsky's slice filtration for logarithmic TC and variants and the BMS motivic filtrations. In the case of perfect fields admitting resolution of singularities and in the étale setting, we show that the slice filtration recovers the BMS motivic filtration, compatibly with the motivic trace map. This is a report on recent joint works with Merici-Lundemo-Park and Park-Østvaer.

Motivic cohomology of mixed characteristic schemes

Thursday
July 11
11:40 -
12:30

Tess Bouis
Université Paris-Saclay, France

I will present a new theory of motivic cohomology for general (qcqs) schemes. It is related to non-connective algebraic K-theory via an Atiyah-Hirzebruch spectral sequence. In particular, it is non-A^1-invariant in general, but it recovers classical motivic cohomology on smooth schemes over a Dedekind domain after A^1-localisation. The construction relies on the syntomic cohomology of Bhatt-Morrow-Scholze and the cdh-local motivic cohomology of Bachmann-Elmanto-Morrow, and generalises the construction of Elmanto-Morrow in the case of schemes over a field.

Cohen-Macaulay motives and perverse homotopy t-structure

Friday
July 12
9:30 -
10:20

Frédéric Déglise
ENS de Lyon, France

Grothendieck's 6 functors formalism was motivated by the theory of dualizing complexes. A crucial part in that play is taken by Cousin complexes and the related notion of Cohen-Macaulay sheaves. On the other side, Beilinson-Voevodsky's motivic sheaves are now fully equipped with a 6 functors formalism, compatible (through realization) with its model on l-adic étale complexes. Though one does not have the motivic t-structure at our disposal, an alternative t-structure has emerged through the works of Voevodsky, Ayoub and Bondarko-Déglise, called the perverse homotopy t-structure. It allows one to define a Leray type spectral sequence and open the way to fibration methods. Whereas the t-structure is reasonably well understood over a field, the perverse homotopy heart was only conjecturally described over a positive dimensional base by Ayoub. In the talk, I will describe a work in collaboration with Niels Feld and Fangzhou Jin which goes beyond this conjecture. We introduce the Cohen-Macaulay property for motivic complexes and show that the associated Cousin motivic complexes embed fully faithfully in the motivic category. Under suitable assumptions, we identify these motivic complexes with (generalized) Rost cycle modules, and with the perverse homotopy heart. Applications to the (perverse homotopy) Leray spectral sequence will be given (if time allows).

Flip-flopping de Rham cohomology along the Lubin-Tate and Drinfeld towers

Friday
July 12
16:40 -
17:30

Gabriel Dospinescu
Ecole Normale Supérieure de Lyon, France

I will explain how general p-adic comparison theorems for rigid analytic varieties allow to prove a comparison between the de Rham cohomology of the Lubin-Tate tower and that of the Drinfeld tower. This is joint work with Guido Bosco and Wieslawa Niziol.

Higher pushforwards in rigid cohomology via motives

Monday
July 8
15:10 -
16:00

Veronika Ertl
IMPAN Warszawa, Poland

Berthelot's conjecture states that the higher push-forwards in rigid cohomology of the structure sheaf along a smooth and proper morphism are canonically overconvergent F-isocrystals. I will explain how motivic non-archimedean homotopy theory can be used to define solid relative rigid cohomology and prove a version of Berthelot's conjecture. (Joint work with Alberto Vezzani.)

Serre's Euler-Poincaré functions and local systems

Tuesday
July 9
11:00 -
11:50

Yuval Flicker
Ariel University, Israel

The Euler-Poincaré function of Serre, which is a pseudo-coefficient of the Steinberg representation in view of computations of continuous cohomology of admissible representations, will be used together with the automorphic trace formula to give a new proof of the count (obtained with P. Deligne) of the equivalence classes of local systems of rank n on a curve X over a finite field $\mathbb{F}_q$, fixed by the Frobenius, with principal unipotent monodromy at least at two points. This EP-function plays a key role in Kottwitz' reduction of Weil's conjecture on Tamagawa numbers over a number field to the quasi-split case, leading to an important open question.

Zagier's conjecture on polylogarithms: from function fields to number fields

Thursday
July 11
16:40 -
17:30

Quentin Gazda
Centre de Mathématiques Laurent Schwartz (CMLS), École Polytechnique, France

Zagier's conjecture is a certain formulation of the following slogan: linear relations among polylogarithms evaluated at algebraic numbers arise from relations between K-theory symbols. By identifying the different attributes of this conjecture, one can state and prove a similar version in arithmetic of function fields. Classical polylogarithms are then replaced by those of Carlitz. The proof, very different from the techniques developed so far, uses ingredients from the theory of difference equations. It involves deformations of Carlitz polylogarithms where a new variable t appears. This results from a joint work with A. Maurischat. While we currently lack the technology to reproduce this argument in number theory, it is amusing to speculate on a hypothetical transcription. q-deformations of polylogarithms then replace these « t-deformations ». With T. Bouis, we recently encountered versions of q-polylogarithms in the syntomic Chern class introduced by Bhatt-Lurie. This is encouraging...! I will mention these works in a second part of the presentation.

The fullness conjectures for products of elliptic curves

Wednesday
July 10
9:00 - 9:50

Bruno Kahn
Institut de Mathématiques de Jussieu-Paris Rive Gauche, Sorbonne Université, France

We give a uniform proof (at least as uniform as possible) for products of elliptic curves of all the conjectures in Chapter 7 of Yves André's book on motives; these conjectures are the Hodge conjecture, the Tate conjecture, the Ogus conjecture and the de Rham-Betti conjecture (also called weak Grothendieck period conjecture). After reducing to a large enough ground field, we prove an abstract fullness statement under an axiomatic framework. Then the axioms are checked case by case. Some of these conjectures were previously known; the present strategy substantially simplifies their proofs.

Extension of Drinfeld modules and Anderson t−modules

Tuesday
July 9
16:40 -
17:30

Dawid Kędzierski
University of Szczecin, Poland

This talk is about joint work with Piotr Krasoń. Our aim is to study the structure of the extension group $\mathrm{Ext}^1_\tau(\phi,\psi)$ in the case when ψ, ϕ are Drinfeld modules or specific classes of $\mathbf{t}-$modules. The main idea is to describe the space of extensions by biderivations, that is, in the language of linear algebra.

The background of this can be found in the Hochschild's thesis on the study of extensions (the first Hochschild cohomology groups) of associative algebras. In the 1970′s C.M. Ringel applied this idea to the case of the extension of modules over path algebras of a quiver or more generally for the $K-$species. In 2003 M. A. Papanikolas and N. Ramachandran introduced the concept of biderivations for $\mathbf{t}-$modules and used this to prove a Weil-Barsotti formula for Drinfeld modules.

We will show that if ϕ and ψ are Drinfeld modules satisfying the condition $\mathrm{rk}\,\phi > \mathrm{rk}\,\psi$, then $\mathrm{Ext}^1(\phi,\psi)$ has the structure of a $\mathbf{t}-$module. We will give a complete algorithm describing this structure. Next we will generalize this to the case where ϕ is a strictly pure $\mathbf{t}-$module and ψ is a Drinfeld module or a tensor power of the Carlitz module. We will also establish duality between Ext groups for $\mathbf{t}-$modules and corresponding adjoint $\mathbf{t}^\sigma-$modules. We will prove the existence of $\mathrm{Hom}-\mathrm{Ext}$ six-term exact sequence for $\mathbf{t}-$modules and dual $\mathbf{t}-$motives. As the category of $\mathbf{t}-$modules and dual $\mathbf{t}-$motives are only additive (not abelian) these results are quite surprising. We will use this to obtain the structure of the $\mathrm{Ext}^1(\Phi,\Psi)$ group when Φ and Ψ admit composition series with Drinfeld modules as sub-factors. Finally, we will present a computer program for determining t-structures on Ext groups, implemented by our student Filip Głoch.

Combinatorial algebraic K-theory

Wednesday
July 10
11:40 -
12:30

Bernhard Koeck
University of Southampton, UK

In 2012, Dan Grayson has come up with an explicit description of higher algebraic K-groups in terms of generators (so-called binary complexes) and relations. In this talk, I'll report on how this description can be used to study exterior power operations on higher algebraic K-theory and how to extend it to relative algebraic K-groups. This is joint work with Tom Harris, Lenny Taelman, Ferdinando Zanchetta and Jane Turner.

Commutative group schemes and multiplicative polynomial laws

Monday
July 8
16:40 -
17:30

Dmitry Kubrak
Institute for Advanced Study, Princeton, USA

I will talk about the joint project with Mathew, Raksit and Zavyalov where (following a suggestion by Scholze) we develop a notion of "group sheaves with multiplicative polynomial laws". Classical group schemes naturally give examples of such objects, but the feature of this new setup is that (at least some) Ext groups behave much nicer than say in the category of flat group sheaves. On the other hand in the derived affine setting this gives a model for derived commutative group schemes where both multiplication and comultiplication on the corresponding Hopf algebra are "animated". I will talk about some basic computations in this new category and expected applications (e.g. that the (derived) moduli space of finite commutative group schemes in this setting is a derived lci).

Characteristic classes of étale local systems

Monday
July 8
11:40 -
12:30

Alexander Petrov
Institute for Advanced Study, Princeton, USA

Given an étale $\mathbb{Z}_p$-local system of rank n on an algebraic variety X, continuous cohomology classes of the group $\mathrm{GL}_n(\mathbb{Z}_p)$ give rise to classes in (absolute) étale cohomology of the variety. These characteristic classes can be thought of as p-adic analogs of Chern-Simons characteristic classes of vector bundles with a flat connection.

For a smooth projective variety over complex numbers, Reznikov proved that the usual Chern-Simons classes in degrees > 1 of all C-local systems are torsion. It turns out that characteristic classes of étale $\mathbb{Z}_p$-local systems on algebraic varieties over non-closed fields are often non-zero even rationally. In particular, if X is a smooth variety over a p-adic field, and the local system is de Rham, then its characteristic classes are related to Chern classes of the graded quotients of the Hodge filtration on the associated vector bundle with connection. Our main tool for studying these characteristic classes is the notion of Chern classes for pro-etale vector bundles on X: they are related to characteristic classes of local systems via a computation of Soule's regulator on (continuous) K-theory of $\mathbb{Z}_p$. This is joint work with Lue Pan.

An Equivariant Main Conjecture in Iwasawa Theory and Applications

Thursday
July 11
9:00 - 9:50

Cristian Popescu
University of California San Diego, USA

I will discuss the statement and proof of an equivariant main conjecture in Iwasawa theory, building upon the recent work of Dasgupta-Kakde on the Galois module structure of the Selmer groups defined by Burns-Kurihara-Sano. I will make connections with my earlier joint results with Greither in geometric Iwasawa theory and give applications to the Galois module structure of the even Quillen K–groups of rings of algebraic integers. This is based on joint work with Rusiru Gambheera.

Nonexistence of projective smooth representations in natural characteristic

Monday
July 8
9:00 - 9:50

Peter Schneider
Universität Münster, Germany

Let G be a p-adic Lie group. We consider the category $\mathrm{Mod}(G)$ of smooth G-representations over a field k of characteristic p. In joint work with C. Sorensen we are interested in the nonexistence of nonzero projective objects in $\mathrm{Mod}(G)$. In this generality we show that this nonexistence holds provided the center of G is non-discrete. Note that such a condition is necessary since for finite G one always has enough projective objects. We have a completely general result, though, if G is the group of F-points of a connected reductive group over a finite extension F of $\mathbb{Q}_p$. Then $\mathrm{Mod}(G)$ has no nonzero projective objects provided G is nontrivial. The underlying technique is the study of derived smooth induction from compact open subgroups of G together, in the reductive case, with a heavy use of Bruhat-Tits theory.

The generalized height pairing via mixed Hodge modules

Wednesday
July 10
10:10 -
11:00

Tamás Szamuely
Università di Pisa, Italy

With D. Rössler we have defined a generalization of Beilinson's geometric height pairing for algebraic cycles on smooth projective varieties defined over the function field of a variety B. Our height pairing has values in the second cohomology of B but conjecturally should come from a geometric pairing with values in the Q-Picard group. Using techniques of mixed Hodge modules, we can produce such a pairing unconditionally in the case when the base field is C. This is joint work in progress with P. Brosnan and G. Pearlstein.

ϵ-isomorphisms for analytic (φ_L, Γ_L)-modules over Lubin-Tate Robba rings

Tuesday
July 9
15:10 -
16:00

Otmar Venjakob
Universität Heidelberg, Germany

Inspired by Nakamura's work on ϵ-isomorphisms for (φ, Γ)-modules over (relative) Robba rings with respect to the cyclotomic theory we formulate an analogous conjecture for L-analytic Lubin-Tate (φ_L, Γ_L)-modules over (relative) Robba rings for any finite extension L of $\mathbb{Q}_p$. In contrast to Kato's and Nakamura's setting, our conjecture involves L-analytic cohomology instead of continuous cohomology within the generalized Herr complex. Similarly, we restrict to the identity components of D_{cris} and D_{dR}, respectively. For rank one modules of the above type or slightly more general for trianguline ones, we construct ϵ-isomorphisms for their Lubin-Tate deformations satisfying the desired interpolation property. This is joint work with Milan Malcic, Rustam Steingart and Max Witzelsperger.

Relative mod-p Poincare Duality in p-adic analytic geometry

Thursday
July 11
15:10 -
16:00

Bogdan Zavyalov
Institute for Advanced Study, Princeton, USA

Motivated by the work of Berkovich, we define a trace morphism for an (almost) arbitrary smooth morphism of rigid-analytic varieties over a p-adic field. As the first application of this construction, we give a new easier (and essentially "diagrammatic") proof of mod-p Poincare Duality for smooth proper rigid-analytic spaces. As another application, we also prove a version of Poincare Duality for an arbitrary proper morphism of rigid-analytic spaces; this positively confirms the expectation put forward by Bhatt and Hansen.

This is a joint work in progress with Shizhang Li and Emanuel Reinecke.

Speakers